Softstuf, Inc.

Power Calculations in AC Circuits

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Power Formula for an Alternating Current Circuit

$P(t) = V(t)I(t)$ (Instantaneous Power)

$$P(t) = V_{Max.} \sin(\omega t + \theta_V)\, I_{Max.} \sin(\omega t + \theta_I) \qquad (1)$$

$Where$:

$\omega = Angular\ Frequency$

$t = Time$

$\theta_V = Phase\ of\ Voltage$

$\theta_I = Phase\ of\ Current$

Using the Product-to-Sum Formula ($\sin u \sin v = (1/2)[\cos(u - v) - \cos(u + v)]$) and, taking $u = \omega t + \theta_V$ and $v = \omega t + \theta_I$ equation (1) can be written as shown below.

$$P(t) = \frac{V_{Max.}\, I_{Max.}}{2} [\cos(\theta_V - \theta_I) - \cos(2\omega t + \theta_V + \theta_I)]$$

To find the average power over one cycle the average value formula must be applied to the equation above.

$$Average\ Value\ Formula: \quad f_{Avg.}(x) = \frac{1}{b - a} \int_a^b f(x)\, dx$$

$$P_{Avg.} = \frac{1}{T} \int_0^T \frac{V_{Max.}\, I_{Max.}}{2} [\cos(\theta_V - \theta_I) - \cos(2\omega t + \theta_V + \theta_I)]\, dt$$

$$= \frac{V_{Max.}\, I_{Max.}}{2T} \left[\cos(\theta_V - \theta_I)\, t - \frac{1}{2\omega} \sin(2\omega t + \theta_V + \theta_I)\right]_0^T$$

$$= \frac{V_{Max.}\, I_{Max.}}{2T} \left[\left(\cos(\theta_V - \theta_I)T - \frac{1}{2\omega} \sin(2\omega T + \theta_V + \theta_I)\right) - \left(0 - \frac{1}{2\omega} \sin(\theta_V + \theta_I)\right)\right]$$

$$= \frac{V_{Max.}\, I_{Max.}}{2T} \left[\cos(\theta_V - \theta_I)T - \frac{1}{2\omega} \sin(2\omega T + \theta_V + \theta_I) + \frac{1}{2\omega} \sin(\theta_V + \theta_I)\right]$$

$$Angular\ Frequency: \quad \omega = \frac{2\pi}{T}$$

$$= \frac{V_{Max.}\, I_{Max.}}{2T} \left[\cos(\theta_V - \theta_I)T - \frac{T}{4\pi} \sin(4\pi + \theta_V + \theta_I) + \frac{T}{4\pi} \sin(\theta_V + \theta_I)\right]$$

$$= \frac{V_{Max.}\, I_{Max.}}{2} [\cos(\theta_V - \theta_I) + 0]$$

$$= V_{RMS} I_{RMS} \cos(\theta_V - \theta_I) \qquad (2)$$

The term $\cos(\theta_V - \theta_I)$ is known as the power factor and is a measure of how much power is dissipated in an AC circuit. Using Euler's Formula ($Acos\varphi + jAsin\varphi = Re(Ae^{j\varphi}) + j\ Im(Ae^{j\varphi})$) equation (2) can be written as shown below.

$$P_{Avg.} = Re(V_{RMS}I_{RMS}e^{j(\theta_V - \theta_I)})$$

This equation shows that the average power represents the real component of power in the complex plane. Complex power, $\boldsymbol{S}$, can now be described in exponential form in which it is equal to the product of the voltage, $\boldsymbol{V}$, and the complex conjugate of the current, $\boldsymbol{I}^*$.

$$\boldsymbol{V} = V_{RMS}e^{j\theta_V}$$
$$\boldsymbol{I}^* = I_{RMS}e^{-j\theta_I}$$
$$\boldsymbol{S} = V_{RMS}I_{RMS}e^{j(\theta_V - \theta_I)}$$

Using Euler's Formula again, the equation above can be written as shown below.

$$\boldsymbol{S} = V_{RMS}I_{RMS}[\cos(\theta_V - \theta_I) + j\ \sin(\theta_V - \theta_I)]$$

This equation shows that power in an AC circuit has both a real and an imaginary component. The real component, $V_{RMS}I_{RMS}\cos(\theta_V - \theta_I)$, is known as active power and the imaginary component, $V_{RMS}I_{RMS}\sin(\theta_V - \theta_I)$, is known as reactive power. Apparent power is equal to the product of the RMS values of the current and voltage.

Summary

$$Active\ Power = V_{RMS}I_{RMS}\cos(\theta_V - \theta_I)$$

$$Reactive\ Power = V_{RMS}I_{RMS}\sin(\theta_V - \theta_I)$$

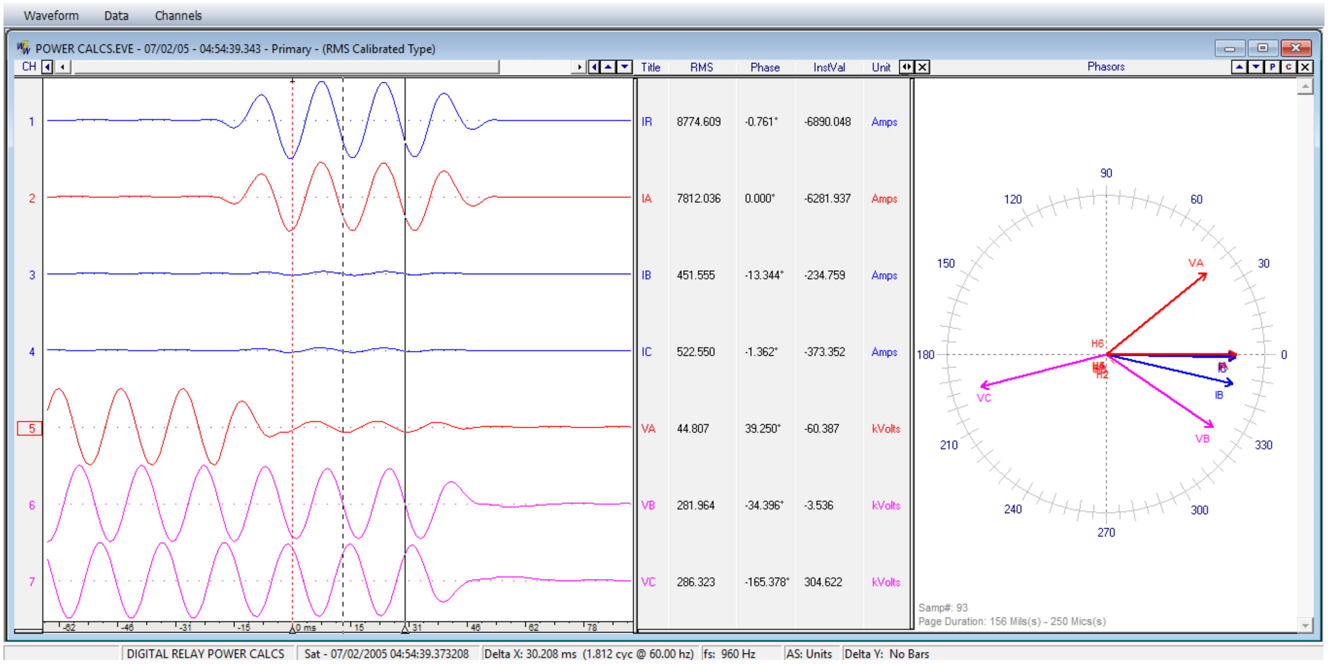
$$Apparent\ Power = V_{RMS}I_{RMS}$$

$$Power\ Factor = \cos(\theta_V - \theta_I)$$

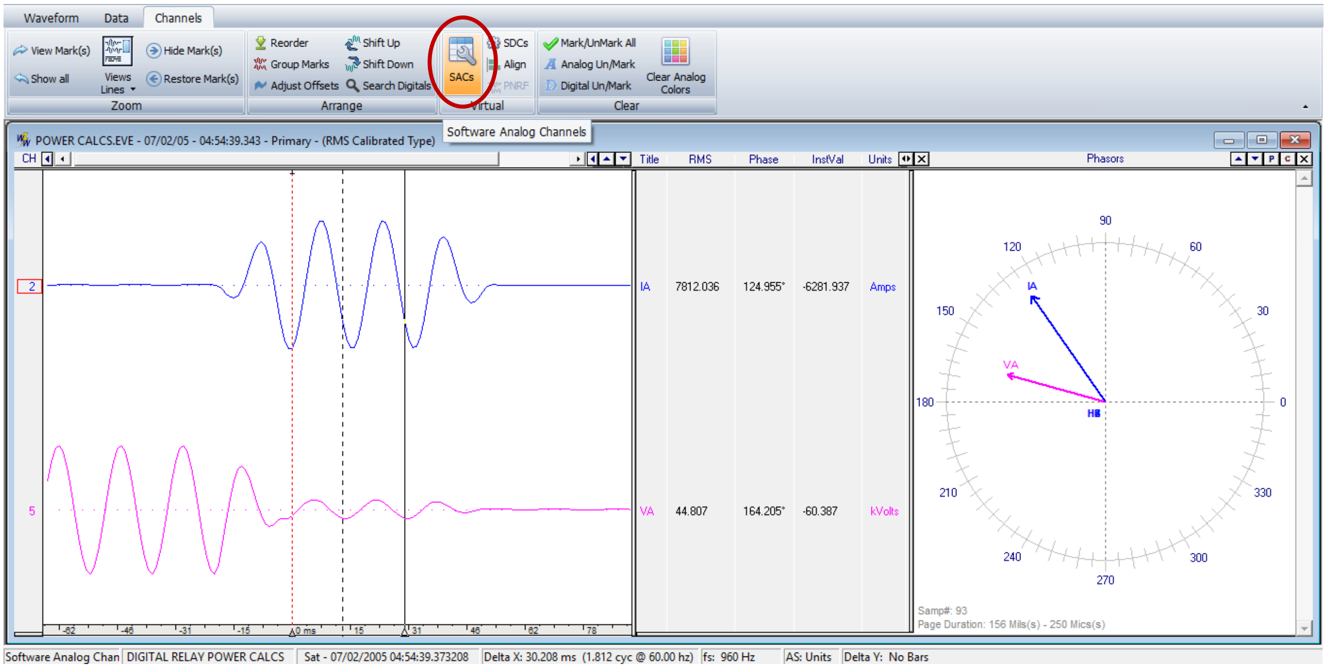
In summary, power has two components: active and reactive. Active power is a measure of how much power is truly dissipated in a circuit whereas reactive power is a measure of how much power is flowing back and forth between the source and the load. The Software Analog Channels in Wavewin can be used to calculate and analyze the four power equations listed above. A short tutorial is shown below.

<u>How to Calculate Power using Wavewin</u>

<u>Step 1:</u> Select a current and voltage waveform and press enter to isolate the two.



Step 2: In the Channels tab click SACs (Software Analog Channels).



Step 3: In the pop-up window click Show Help for a summary of how to use the Analog Channels. In short, the Operators column is used to write equations and the Titles column is used to name them. The SACs read reverse polish notation. Using this notation, the equations for power (where channel 5 represents the voltage and channel 2 represents the current) should be written as follows.

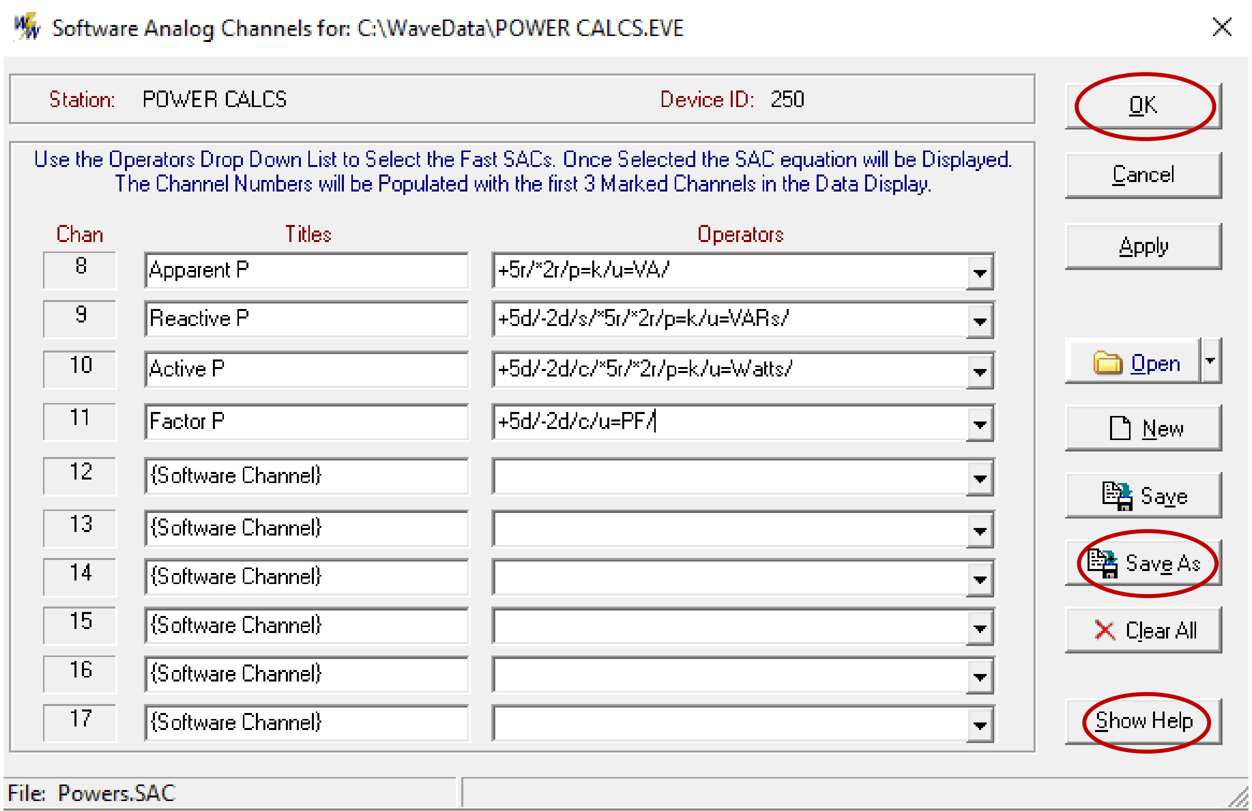
Apparent Power = +5r/*2r/p=k/u=VA/

Reactive Power = +5d/-2d/s/*5r/*2r/p=k/u=VARs/

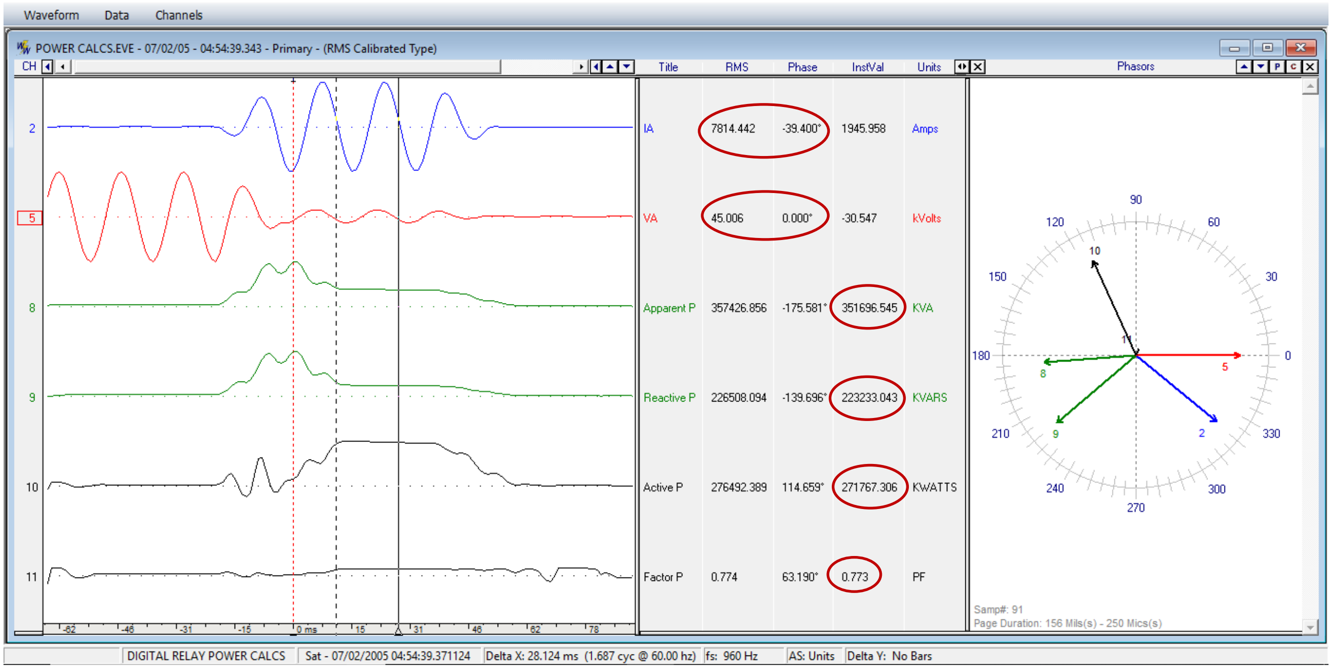
Active Power = +5d/-2d/c/*5r/*2r/p=k/u=Watts/

Power Factor = +5d/-2d/c/u=PF/

Once all the equations are written they can be saved for further use using the Save As button then, click OK to execute.



Step 4: Use the Analog Table to read the Instantaneous values.



Accuracy Calculations

The instantaneous and phase values from the image above can be used to find a percent error between the theoretical values of power and those calculated by Wavewin. The image above shows the RMS and Phase values to be:

Voltage: RMS = 45.006 kV   Phase = 0.000°
Current: RMS = 7814.442 A   Phase = -39.400°

Using the above data, the theoretical values of power are as follows.

Active Power = 271767.907 kWatts
Reactive Power = 223232.676 kVARs
Apparent Power = 351696.777 kVA
Power Factor = 0.773

Therefore, the power calculated in Wavewin comes within 0.0003% error as compared to the theoretical values.

Note: Due to the way equations are calculated in the SACs, power factor will only be positive if $\cos(\theta_V - \theta_I)$ is positive. However, using the IEEE Power Factor Sign Convention, power factor is positive when active and reactive power have different signs (i.e. + and -). If the signs are similar (+ and + or - and -) then power factor is negative. Also, if active power is positive then power is being delivered to the load, if it is negative then power is being sent back to the source. In the example above the active and reactive power have similar signs (+) therefore the power factor should be -0.773 and it is being delivered.